

Joule Thomson co-efficient and Inversion Temperature

Introduction:

In thermodynamics, the effect called Joule Thomson effect was discovered in 1852. This effect was named after two physicists James Prescott Joule and William Thomson. This effect describes the change in temperature of a real gas or liquid when it is forced to pass through porous plug or valve while it is kept insulated so that exchange of heat with the environment is not possible. This process is called throttling process or Joule Thomson. This effect is mainly used in thermal machines such as refrigerators, heat pumps and air conditioners.

Joule Thomson Effect:

Let the stream of gas which is at high pressure is allowed to expand by passing through porous plug into region of low pressure or vacuum. The gas gets cool under adiabatic conditions. But hydrogen and helium are exceptions as they get warmed up under similar circumstances.

The temperature below which a gas becomes cooler on expansion is known as the inversion temperature. For example, the inversion temperature of hydrogen is -48°C while that of helium is -242°C .

Thus **Joule-Thomson effect can be defined as the phenomenon of temperature change produced when a gas is allowed to expand adiabatically from a region of high pressure to a region of extremely low pressure.**

This cooling of the gas is basically due to the decrease in the kinetic energy of the gaseous molecules as some part of this kinetic energy is utilized in overcoming intermolecular van der Waals force of attraction during expansion. The Joule-Thomson effect is very small when a gas approaches ideal behavior. Thus for an ideal gas, Joule Thomson effect is zero. Therefore, no absorption or evolution of heat takes place when expansion of an ideal gas occurs i.e. $q = 0$. This is because in an ideal gas, there are no intermolecular van der Waals forces of attraction thus there is no expenditure of energy in overcoming these forces during expansion. Further, when expansion of ideal gas occurs in vacuum then no work is done since pressure against which it expands is zero. From the first law equation we can say that ΔU is also equal to zero. Thus, when an ideal gas undergoes expansion under adiabatic conditions in vacuum, no change takes place in its internal energy. In other words, the internal energy of a given quantity of an ideal gas at a constant temperature is independent of its volume, i.e.,

$$(\partial U / \partial V)_T = 0.$$

An ideal gas may, therefore, be defined thermodynamically by the following two equations:

(i) $PV = \text{constant}$, at constant temperature

(ii) $(\partial U / \partial V)_T = 0$

The quantity $(\partial U / \partial V)_T$ is called the internal pressure and it is zero for ideal gas.

Joule Thomson Coefficient ($\mu_{J,T}$)

Joule and Thomson derived the relationship between fall of pressure of gas on expansion and resulting lowering of temperature by performing the following technique:

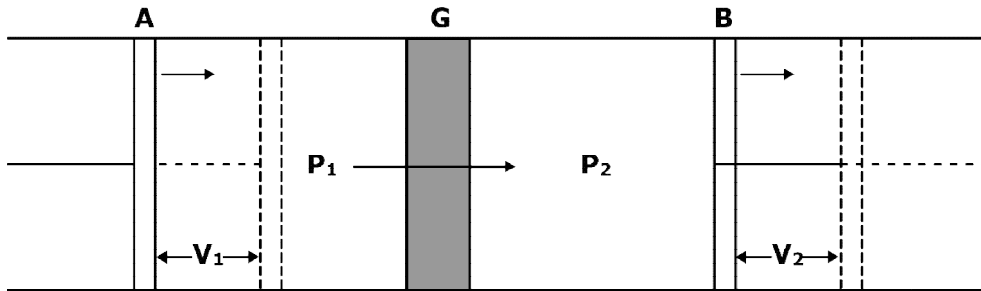


Fig. 1 The porous experiment

A tube made of a non-conducting material is fitted with a porous plug G in the middle and two pistons A and B on the sides, as shown. The tube is thoroughly insulated to ensure adiabatic conditions.

Let the volume of gas enclosed between the piston A and the porous plug G at pressure P_1 is V_1 . This volume is forced to pass through porous plug by moving the piston A inwards. At the same time the volume of gas enclosed between porous plug G and piston B, i.e. V_2 is allowed to expand at a lower pressure P_2 by moving the piston B outward, as shown.

Therefore, work done on the system at the piston A = $+P_1V_1$ and work done by the system at the piston B = $-P_2V_2$

Then, net work done by the system = $-P_2V_2 + P_1V_1$

Since the expansion of the gas is done adiabatically i.e. no exchange of heat takes place between the system and surroundings. Thus the work is done by the system at the expense of internal energy only. Let the internal energy of the system changes from U_1 to U_2 .

$$-P_2V_2 + P_1V_1 = U_2 - U_1 \quad \dots(1)$$

$$U_2 + P_2V_2 = U_1 + P_1V_1 \quad H_2 = H_1$$

$$H_2 - H_1 = 0$$

$$\text{i.e. } \Delta H = 0 \dots\dots\dots (2)$$

This states that the Joule-Thomson expansion of a real gas occurs with constant enthalpy and not with constant internal energy. According to this the process is known as **isoenthalpic process**.

Taking enthalpy as the function of temperature and pressure, then the total differential of the enthalpy H can be written as

$$dH = (\partial H / \partial P)_T dP + (\partial H / \partial T)_P dT \dots(3)$$

$$\text{But } (\partial H / \partial T)_P = C_p \dots(4)$$

Substituting equation (4) in (3) we get

$$dH = (\partial H / \partial P)_T dP + C_p dT$$

Since for adiabatic expansion, $dH = 0$, hence

$$(\partial H / \partial P)_T dP + C_p dT = 0$$

$$\text{Rearranging the above equation } dT/dP = (\partial H / \partial P)_T / C_p \dots(5)$$

$$\text{i.e., } (\partial T / \partial P)_H = - (\partial H / \partial P)_T / C_p \dots(6)$$

The quantity $(\partial T / \partial P)_H$ is called Joule-Thomson coefficient and it is denoted as $\mu_{J.T}$.

For small pressure range the equation (6) can also be written as:

$$\Delta T = - \frac{(\partial H / \partial P)_T}{C_p} \Delta P \dots(7)$$

In the above equation ΔT is the fall of temperature and ΔP is fall of pressure.

Joule-Thomson Coefficient for an Ideal Gas.

Since $H = U + PV$, substituting this in equation (6) we get,

$$\begin{aligned}(\partial T / \partial P)_H = \mu_{J,T} &= - \frac{[\partial(U+PV) / \partial P]_T}{C_p} \\ &= - \frac{\left(\frac{\partial U}{\partial P}\right)_T + \left(\frac{\partial PV}{\partial P}\right)_T}{C_p} \\ &= - \frac{\left(\frac{\partial U}{\partial P} \times \frac{\partial V}{\partial P}\right)_T + \left(\frac{\partial PV}{\partial P}\right)_T}{C_p} \quad \dots(8)\end{aligned}$$

As the internal energy of an ideal gas at constant temperature is independent of its volume i.e.

for an ideal gas, $(\partial U / \partial V)_T$ is zero, therefore

$$(\partial U / \partial V)_T (\partial V / \partial P)_T = 0 \quad \dots(9)$$

Also, PV is independent of temperature in the case of ideal gas i.e.

$$\{\partial(PV) / \partial P\}_T = 0$$

Hence eq. (8) becomes $\mu_{J,T} = 0$.

This shows that the Joule-Thomson coefficient for an ideal gas is zero.

Joule-Thomson Coefficient for a Real Gas

There is no Joule Thomson effect for ideal gas but real gases do have Joule Thomson effect.

So we can conclude that the Joule Thomson effect depends internal energy is dependent on change in volume. $(\partial U / \partial V)_T$ is positive for real gas. This can be explain as follows:

Consider the expansion of real gas in vacuum. In such expansion no external work is done since the external pressure is zero. But some work will definitely be done in separating the gas molecules against the intermolecular forces of attraction (the van der Waals forces). This work is stored in the gas in the form of potential energy,

In other words, the potential energy of the gas increases and hence kinetic energy decreases by an equivalent amount if no heat is exchanged between the system and the surroundings.

Due to this temperature of the gas falls.

Derivation of Joule Thomson coefficient and Inversion Temperature:

The Joule-Thomson coefficient can be calculated with the help of the van der Waals equation.

Considering the van der Waals equation, i.e.

$$(P + \frac{an^2}{V^2})(V - nb) = nRT \dots\dots\dots (10)$$

$$(PV + \frac{an^2}{V} - Pnb - \frac{abn^3}{V^2}) = nRT$$

In this equation, both terms a and b are small, hence the term $\frac{ab}{V^2}$ can be neglected when the pressure of the system is low. By rearranging this equation for 1 mole and neglecting the term $\frac{ab}{V^2}$ we get,

$$PV = RT - \frac{a}{V} + bP \dots(11)$$

Substituting V by RT/P (approximation) we have

$$PV = RT - \frac{aP}{RT} + bP \dots(12)$$

Dividing the whole equation by P, we get

$$V = \frac{RT}{P} - \frac{a}{RT} + b \dots\dots\dots (13)$$

Differentiating with respect to temperature at constant pressure, we get

$$(\frac{\partial V}{\partial T})_P = \frac{R}{P} + \frac{a}{RT^2}$$

Rearranging Eq. 12, we have,

$$RT = P(V - b) + \frac{aP}{RT} \dots(15)$$

Dividing both sides of the above equation by PT, we get

$$\frac{R}{P} = \frac{V - b}{T} + \frac{a}{RT^2} \dots(16)$$

Replacing the value of R/P from Eq. 16 in Eq. 14, we have

$$(\frac{\partial V}{\partial T})_P = \frac{V - b}{T} + \frac{a}{RT^2} + \frac{a}{RT^2} \dots\dots\dots (17)$$

$$T(\frac{\partial V}{\partial T})_P - V = \frac{2a - b}{RT} \dots(18)$$

Now using the thermodynamic relation

$$V = T \left(\frac{\partial V}{\partial T} \right)_P + \left(\frac{\partial H}{\partial P} \right)_P \quad \dots(19)$$

Then Eq. 18 may be written as:

$$-\left(\frac{\partial H}{\partial P} \right)_T = \frac{2a}{RT} - b \quad \dots \quad (20)$$

From Equations (6) and (20) we have

$$\left(\frac{\partial T}{\partial P} \right)_H = \frac{1}{C_P} \left[\frac{2a}{RT} - b \right] \quad \dots\dots(21)$$

From the above expression we conclude that Joule Thomson coefficient will be positive when $2a/RT$ is greater than b . It will become zero when $2a/RT$ is equal to b and negative if $2a/RT$ is less than b . Since a , b and R are constants, therefore magnitude and sign of Joule Thomson coefficient will be dependent only on the temperature at which the expansion of the gas occurred.

The temperature at which the sign of the Joule-Thomson coefficient changes is known as the inversion temperature. At this temperature, $\mu_{J,T}$ is zero. Thus equation (21) will be zero when term inside the parenthesis will be zero so we get,

$$2a / RT_i = b \text{ or } T_i = 2a / Rb \quad \dots(22)$$

where T_i stands for the inversion temperature. Thus, the inversion temperature is dependent on the van der Waals constants a and b of the gas.

$$\mu_{J,T} = \frac{1}{C_P} \left[\frac{2a}{RT} - b \right]$$

Summary:

- The temperature below which a gas becomes cooler on expansion is known as the **inversion temperature**.
- Thus **Joule-Thomson effect** can be defined as the phenomenon of temperature change produced when a gas is allowed to expand adiabatically from a region of high pressure to a region of extremely low pressure.
- Joule-Thomson expansion of a real gas occurs with constant enthalpy and not with constant internal energy and thus is also known as isenthalpic process.
- Joule Thomson coefficient is given by $(\partial T / \partial P)_H = - (\partial H / \partial P)_T / C_P$
- Joule Thomson coefficient for ideal gas is zero
- Joule Thomson coefficient for real gas is given by:

$$\left(\frac{\partial T}{\partial P} \right)_H = \frac{1}{C_P} \left[\frac{2a}{RT} - b \right]$$