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**3 (Sem-4/CBCS) STA HC 2**

**2023**

**STATISTICS**

(Honours Core)

Paper : STA-HC-4026

**(Linear Models)**

Full Marks : 60

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions as directed :

1×7=7

(a) In regression analysis, the variable that is being predicted is

(i) the independent variable

(ii) the dependent variable

(iii) usually denoted by  $x$

(iv) usually denoted by  $r$

(Choose the correct option)

Contd.



(b) The coefficient of determination is

- (i) equal to zero
- (ii) the ratio of explained and total variation
- (iii) usually less than zero
- (iv) 100% of  $(1 - r^2)$

*(Choose the correct option)*

(c) In least square estimation, which of the following is not a required assumption about the error term?

- (i) The expected value of the error term is one
- (ii) The variance of the error term is the same for all values of  $x$
- (iii) The values of the error term are independent
- (iv) The error term is normally distributed

*(Choose the correct option)*

(d) If the regression equation is equal to

$Y = 23.6 - 54.2X$ , then 23.6 is the \_\_\_\_\_ while - 54.2 is the \_\_\_\_\_ of the regression line.

- (i) slope, intercept
- (ii) slope, regression coefficient
- (iii) intercept, slope
- (iv) radius, intercept

*(Choose the correct option)*

(e) Analysis of variance is a statistical method of comparing the \_\_\_\_\_ of several populations.

- (i) standard deviations
- (ii) variances
- (iii) means
- (iv) None of the above

*(Choose the correct option)*



- (f) The sum of squares due to \_\_\_\_\_ measures the variability of the observed values around their respective treatment means
- (i) treatment
  - (ii) error
  - (iii) interaction
  - (iv) total
- (Choose the correct option)
- (g) All OLS estimators are linear estimators.  
(Write True or False)

2. Answer the following questions briefly :  
2×4=8

- (a) State some applications of the analysis of variance.
- (b) What do you understand by components of variation ?
- (c) Define estimability of linear parametric functions.
- (d) Define  $R^2$  in the context of a linear model.

3. Answer **any three** of the following questions :  
5×3=15

- (a) What is a linear model ? Discuss different types of linear models.
- (b) A sample of 20 observations on  $X$  and  $Y$  gave the following data :
 
$$\begin{aligned} \sum Y &= 21.9 & \sum (Y - \bar{Y})^2 &= 86.9 \\ \sum X &= 186.2 & \sum (X - \bar{X}) &= 215.4 \\ \sum (X - \bar{X})(Y - \bar{Y}) &= 106.4 \end{aligned}$$

Estimate the regression equation of  $Y$  on  $X$  and  $X$  on  $Y$ .
- (c) Consider the one-way AOV model  
 $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ , for  $i = 1, 2$  and  $j = 1, 2, 3$   
 Examine if  $\mu, \alpha_1, \alpha_2$  are estimable without any constraints.
- (d) In what respects do AOV, regression analysis and AOCCOV differ ? Discuss briefly.
- (e) Write a note on the technique of hypothesis testing in case of simple regression models.



4. Answer **either** (a) **or** (b) : 10

(a) State and prove the Gauss-Markov theorem.

(b) What is analysis of variance (AOV) ? What are the basic assumptions associated with it ? What are the remedies, if the assumptions are violated ?

5. Answer **either** (a) **or** (b) : 10

(a) Define a linear regression model. Write the basic assumptions of the linear model. Estimate the parameters of the model.

(b) Give linear model (fixed effect) for two-way classification (one observation per cell) and state its assumptions. Derive the analysis of variance of two-way classification through the method of least squares.

6. Answer **either** (a) **or** (b) : 10

(a) Using the following data

Y :	65	57	57	54	66
X :	26	13	16	-7	27

estimate the regression line  $Y = \alpha + \beta X$ , test the hypothesis that  $\beta = 0$  against the alternative  $\beta < 0$  at 5% level of significance, also construct 95% confidence interval for  $\beta$ .

(Given  $t_{0.05,3} = 2.353$ )

(b) Derive the 'analysis of covariance' for a one-way layout (with one consistent variable only).